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Reynolds Stress Closure in Jet Flows Using Wave Models

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Introduction

The initial stages of the research program have concentrated in three areas. In the first a wave model has been developed for the two-dimensional shear layer. This configuration is being used as a test case for the closure schemes. Secondly, numerical methods are under development to solve the non-separable Rayleigh equation. A model problem is being used to assist in the algorithm development. Thirdly, an analytic solution of the Rayleigh equation for a basic elliptic flow has been obtained. This will be used to verify the stability codes developed for arbitrary geometries. Other numerical methods for solving the Rayleigh equation based on the boundary element technique are being examined. These activities are described briefly below and in the attached papers.

Turbulence Closure in a Mixing Layer

A turbulence closure scheme using a wave model has been applied to an incompressible two-dimensional free mixing layer as a test case. In the future, extensions of this model will be used to predict the mean velocity and temperature fields in circular and non-circular jets and describe the characteristic frequency and wavelength properties of the fluctuating flow field.

In the present model the Reynolds stress of the free mixing layer is determined by the characteristics of the most unstable mode of instability driven by the mean flow. At present only a local model has been developed. This will be incorporated subsequently into a numerical solution of the turbulent boundary layer equations. Thus the mean velocity profile is assumed to be known locally. We have based the profile on the experimental data of Patel ref. 1. A new analytic curve fit to the data has been obtained that improves upon Patel's. This is shown in figure (1).

Since the instability mechanism is dominantly inviscid for high Reynolds number free shear flows, the inviscid instability equation, the Rayleigh equation, is solved. A spatial stability analysis has been performed since the large scale structures develop spatially and are best represented in this manner. The methods proposed by Bridges and Morris ref. 2 to solve an eigenvalue problem which is nonlinear in its parameter have been applied successfully. The eigenvalue spectrum is well predicted using the linear companion matrix method. The eigenvalue spectrum of the free mixing layer is shown in figure (2). Another algorithm based on the matrix factorization method enables the most unstable wave mode to be determined. The Reynolds stress distribution based on the characteristics of the most unstable mode is shown in figure (3). It should be noted that the maximum value is based on the experimental data and neglects any contributions from the small-scale turbulence. The Reynolds stress and hence the turbulence production is negative at the outer edge of the mixing region. This has been observed in the conditionally averaged measurements by Komori and Ueda ref. 3. They attributed this phenomenon to the negative production of Reynolds stress by the pressure-strain correlation of the large-scale structure. The present model allows us to calculate such properties directly and this will be pursued in the next stage of the work. However, it is clear from figure (3) that the contribution from the small-scale components of the turbulence must be included in the model. Modeling the relative importance of the small and large scales will depend heavily on experimental data. In the next stage of this work it is intended to establish a close link with the multiple scales approaches to Reynolds stress closure and to incorporate the resulting models in a turbulent boundary layer solver.

The details of the analysis have been used as the basis of a Master's dissertation by Mr. Liou who is supported by the present grant. The title page and abstract are attached. A presentation of these results is to be made at the Meeting of the Fluid Dynamics Division of the American Physical Society in November 1986.

Solution of the Non-Separable Rayleigh Equation

The first step in the solution of the Reynolds-averaged compressible equations of motion for jets of arbitrary geometry using a wave model is the description of the hydrodynamic stability of such flows. This requires the solution of a non-separable form of Rayleigh equation. The most unstable eigensolutions may then be used to model the Reynolds stress associated with the large-scale structures.

During the first six months of this program a method has been developed to determine the eigensolutions of the Rayleigh equation in flows of arbitrary geometry. The equation to be solved is:

$$(\Delta - \alpha^2)\hat{p} + (2\alpha/\Omega)\nabla W \cdot \nabla \hat{p} = 0 \tag{1}$$

with boundary conditions:

$$\hat{p} \quad \text{is finite and} \quad \hat{p} \to 0 \quad \text{at infinity}$$

where $\hat{p}$ is the pressure fluctuation, $W(x,y)$ is the axial mean velocity, α is the axial wavenumber (the complex eigenvalue), ω is the wave frequency, and $\Omega = \omega - \alpha W$.

In order to test various numerical algorithms for solving this problem a model problem with a known analytic solution has been posed. The boundary value problem is given by:

$$\Delta\phi - 2\alpha\omega(\partial_x\phi + \partial_y\phi) - 2\alpha^2\phi = 0 \tag{2}$$

with,

$$\phi = 0 \quad \text{on} \quad \partial\Omega$$

where

$$\Omega := \{(x,y) \ \in \ \mathbf{R}^2 | -1 \leq x, y \leq -1\}$$

The method used to determine the eigenvalues α is a *spectral method* using a two-dimensional Chebyshev series approximation of the form:

$$\phi(x,y) = \sum\sum \phi_{mn} T_n(x) T_m(y). \tag{3}$$

Here the summations are taken from 0 to N.

The determination of α goes as follows: i) The P.D.E. is integrated to eliminate all derivatives of the dependent variable. ii) This integral equation is discretized using the Chebyshev series, equation (3). iii) The boundary conditions are discretized. iv) Steps ii) and iii) reduce to the problem of determining all α such that:

$$\det[\mathbf{A}\alpha^2 + \mathbf{B}\alpha + \mathbf{C}] = 0 \tag{4}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are $(N+1)^2 \times (N+1)^2$ matrices depending on the discretization. v) Equation (4) is solved using globally or locally convergent schemes.

Both locally and globally convergent schemes have been developed and applied to the model problem. Preliminary results suggest that both methods should be used in conjunction with each other to determine the eigenvalues. The globally-convergent scheme used, the companion matrix method, requires the matrices in equation (4) to be very large in order to achieve reasonable accuracy. This is expensive computationally. In contrast the local scheme, based on the Newton-Raphson method gives better accuracy in considerably less time. However the Newton-Raphson method requires a good initial guess for the eigenvalue. Therefore, the best results have been obtained when the eigenvalues have been approximated with the globally-convergent scheme and the accuracy of individual eigenvalues then improved with the local scheme.

In the next stage of this analysis the methods that have been developed for the model problem will be applied to the Rayleigh equation. The way that the boundary conditions are to be applied is yet to be determined. In order to obtain higher accuracy the analytic forms of solution in the potential core region of the jet and outside the jet may be applied at the edges of the mixing layer. Though this reduces the extent of the region in which the partial differential problem must be approximated it means that the globally-convergent scheme may no longer be used. This is because the eigenvalue problem cannot be written in the polynomial form as in equation (4). Once this problem has been resolved the Rayleigh equation will be solved for the circular jet and elliptic jet. The numerical solutions are already available for these problems since the Rayleigh equation is then separable in the appropriate coordinate system. The elliptic jet case has been examined as part of this research program and details are given below.

Instability of Elliptic Jets

The hydrodynamic stability characteristics of an elliptic jet flow has been examined. The results will be used to evaluate the accuracy of the methods for jets of arbitrary cross-section. Details of this analysis are contained in AIAA Paper No. 86-1868 which was presented at the AIAA 10th Aeroacoustics Conference in Seattle, Washington, July 9-11, 1986. A copy of the paper is attached.

In order to render this problem in separable form it necessary to require that the mean velocity is a function of only one coordinate: in this case the "radial" coordinate in an elliptic cylindrical coordinate system. This provides a reasonable match with experimental data close to nozzle exit for thin initial boundary layers. Further downstream there is no reason why the mean velocity should be restricted in this way. That is why the general method for arbitrary geometries is being developed. However the calculations provide a highly accurate test case for such methods.

Additional Activities

Alternative methods are being examined to solve the non-separable Rayleigh equation. These are based on the boundary element technique. In such an approach the differential eigenvalue problem is transformed to an integral form where the integral lies on the boundary between separate regions. Such regions are the potential core of the jet, its mixing layer, and the ambient fluid. When the mixing layer is represented by a vortex sheet all the calculations are performed at the vortex sheet. Even when the mixing layer is of finite thickness the boundary element technique should offer substantial savings over other integral methods such as the finite element method. One result of the analysis, and this is also true of the methods already being developed, the zero frequency solutions of the eigenvalue problem enable the shock structure in the jet to be determined. This has been demonstrated for the circular jet by Tam, Jackson and Seiner ref. 4.

References

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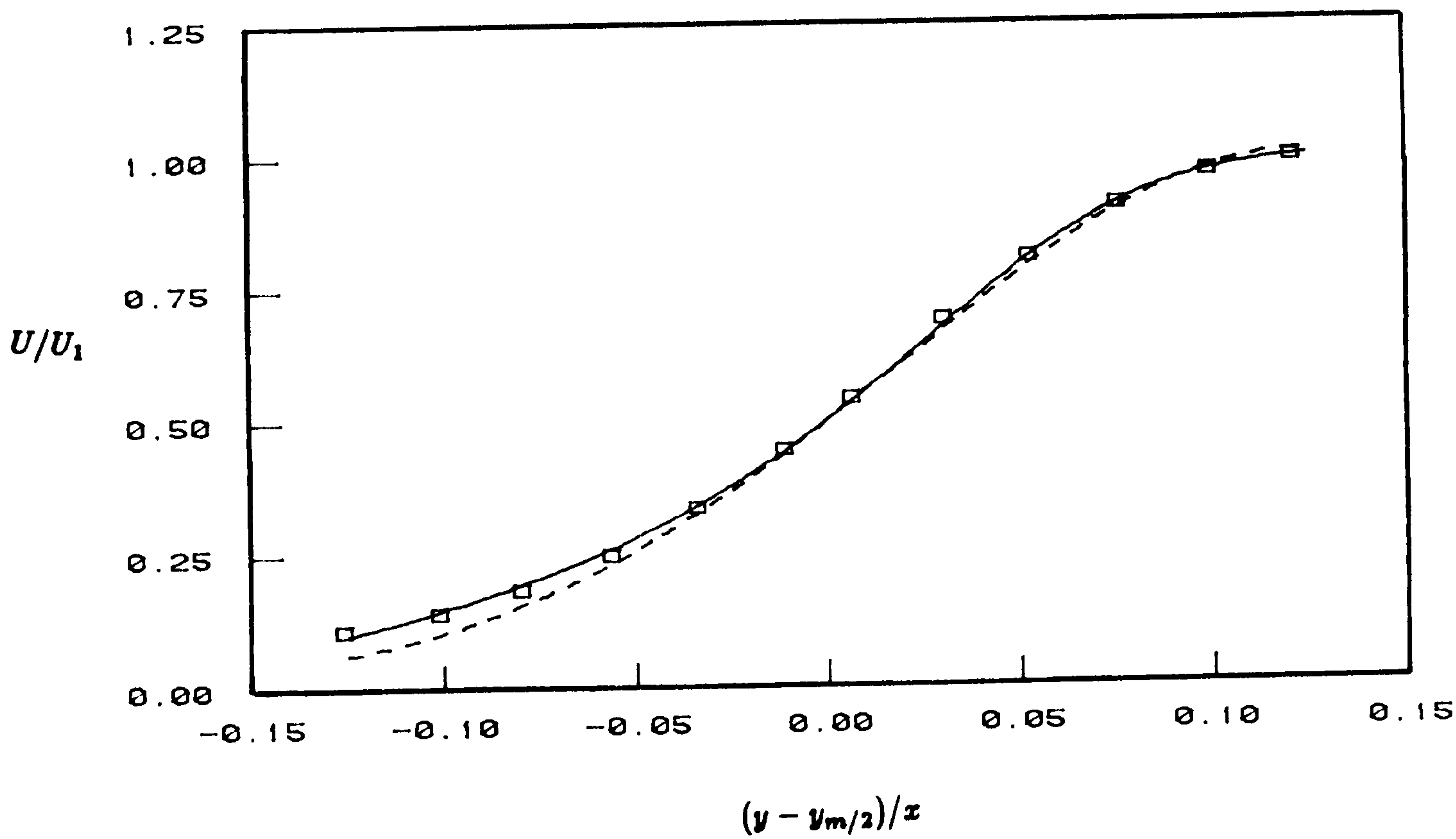

Figure 1. Mean Velocity Profile for the Free Mixing Layer.

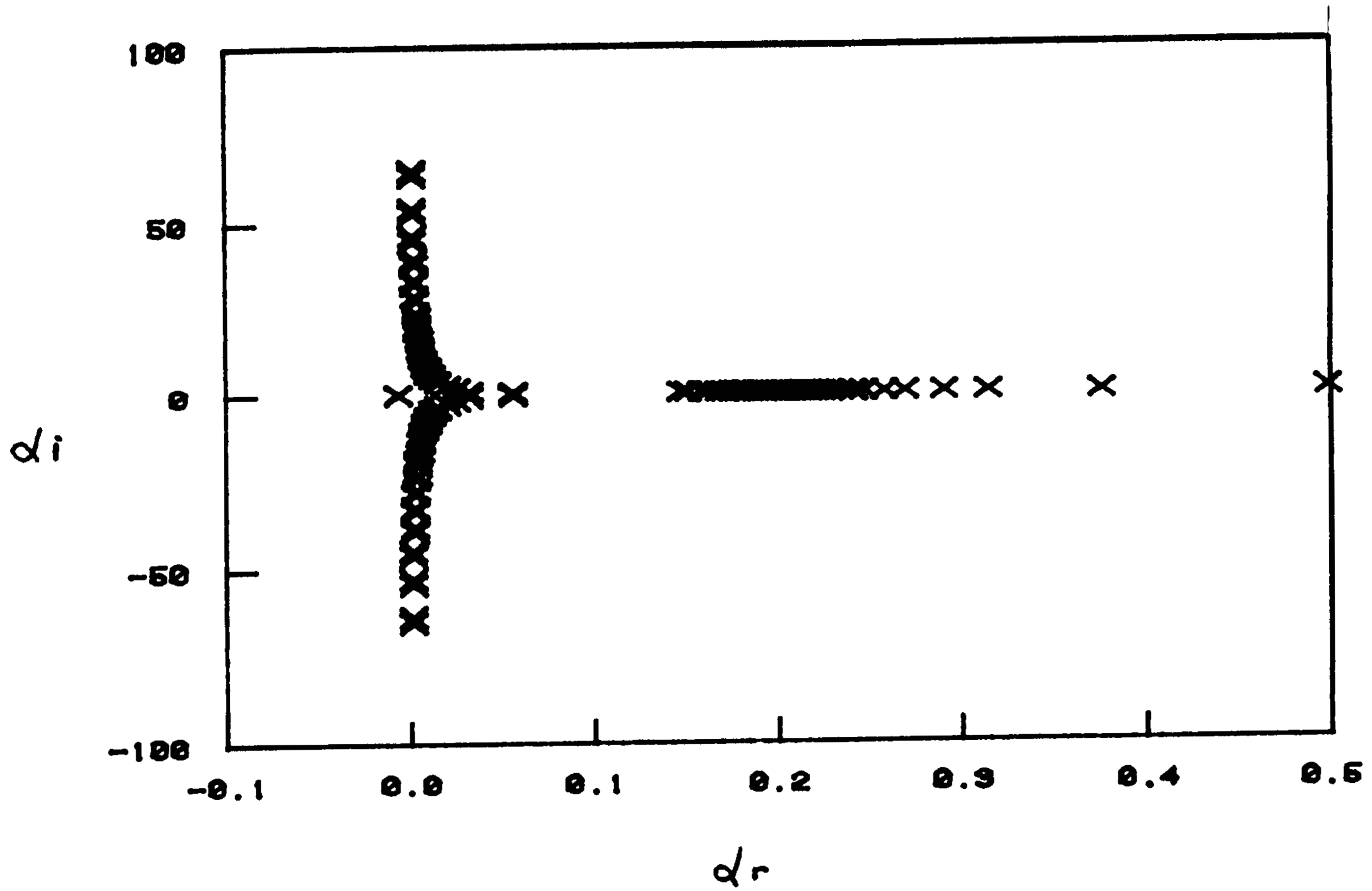

Figure 2 The Eigenvalue Spectrum of the Free Mixing Layer, w=.14.

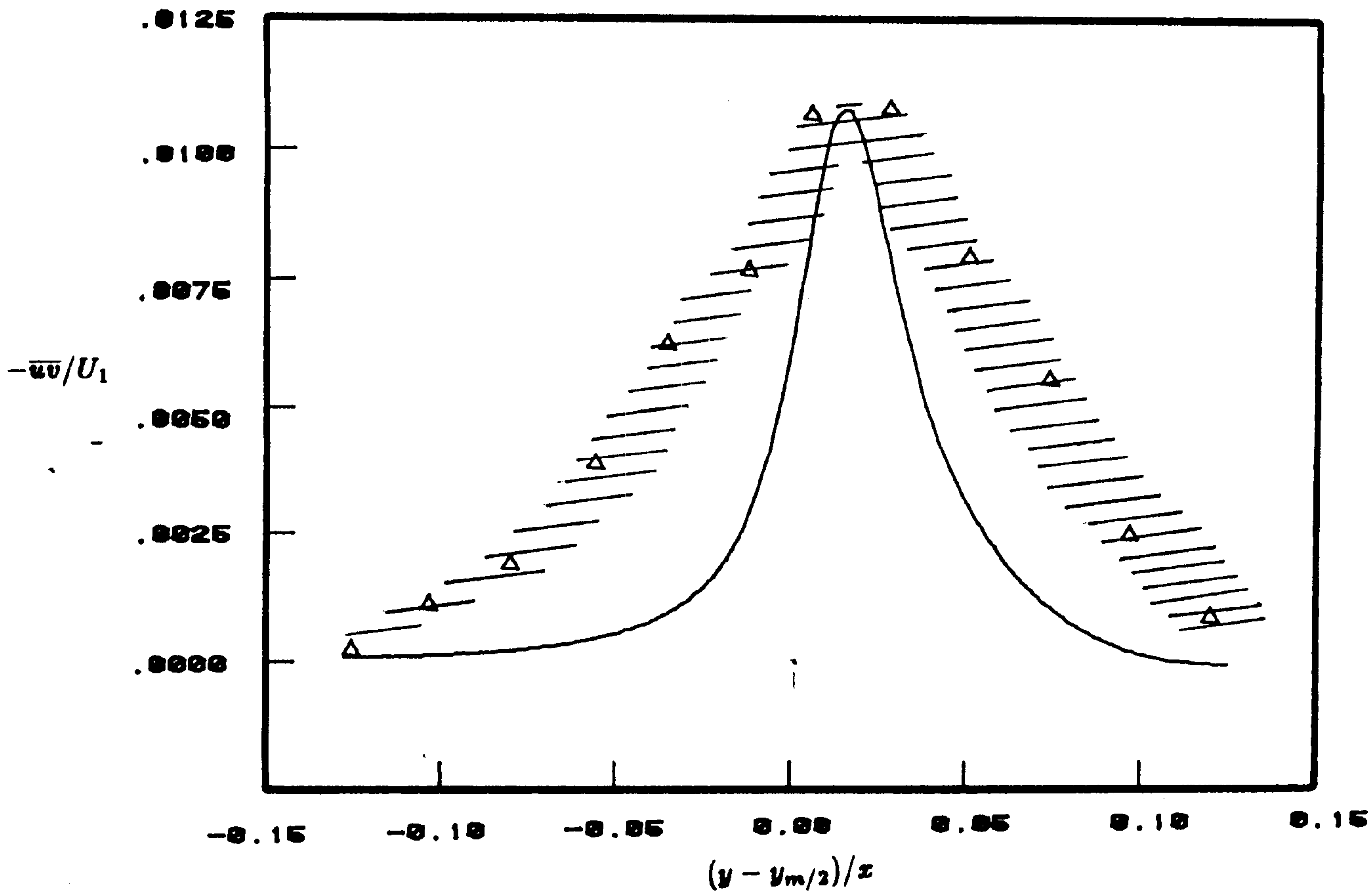

Figure 3 Comparison of Reynolds Stress,

The Pennsylvania State University

The Graduate School

Department of Aerospace Engineering

The Computation of Reynolds Stress in an Incompressible Plane Mixing Layer

A Thesis in

Aerospace Engineering

by

Whai-Wai Liou

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ABSTRACT

The turbulent Reynolds stress of a plane mixing layer is modeled by a wave model in this thesis. The wave model interprets the large scale turbulent structures, which dominate the motion of the flow, as instability wave trains. The Rayleigh equation is solved to provide a description of the spatially amplified instability waves of the flow. The eigenfunction of the Rayleigh equation is approximated by a finite Chebyshev series. The newly developed methods of Bridges and Morris (1984) is applied to solve the resulting nonlinear eigenvalue problem. A detailed analysis of the eigenvalue spectrum is performed. The Reynolds stress thus obtained may then be used for the closure of the turbulent flow equations.